

# SOME CONTRAPUNTAL OBSERVATIONS ON THE MYSTIC CHORD AND SRIABIN'S PIANO SONATA NO. 5

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**ABSTRACT.** We present statistical evidence for the importance of the “mystic chord” in Scriabin’s Piano Sonata No. 5, Op. 53, from a computational and mathematical counterpoint perspective; more specifically, we compute the effect sizes and  $\chi^2$  tests with respect to the distributions of counterpoint symmetries in the Fuxian and mystic counterpoint worlds in two passages of the work, which provide evidence of a qualitative change between them.

## 1. INTRODUCTION

A prominent chord in Alexander Scriabin’s late work is the so-called “mystic chord” or “Prometheus chord”, whose pc-set when the root is C is  $M = \{0, 6, 11, 4, 8, 2\}$  [2, p. 23]. It can be seen a chain of major thirds, and thus can be covered by an augmented and a diminished triad; it is also covered partially by a minor and a major triad, leaving only the root outside. Surely this evidences the strong tonal ambiguity of the chord, which is also associated to the Impressionism in music during the late 19th and early 20th centuries in Europe. It can also be seen as an extension of the French sixth, which is completely contained in a whole tone scale; the mystic chord also has this property safe for a “sensible” tone. As we will see, this is an important feature from the perspective of the mathematical counterpoint theory developed by the second author.

More specifically, the mystic chord is also what is called a *strong dichotomy*, this is, a bipartition of the scale modulo octave such that its only affine symmetry is the identity. In particular, it belongs to the class 78 in Mazzola’s classification as it appears in his *Topos of Music* [4]. A strong dichotomy can be understood as a division of pitch classes in consonances and dissonances. The classical consonances of Renaissance counterpoint also conform a strong dichotomy, and also other five bipartitions (modulo affine symmetries).

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If we study the predicted allowed steps for a counterpoint distilled from  $M$ , we find that a particularly favorable scale is one particular transposed mode the whole-tone scale, namely the one with pc-set  $\{1, 3, 5, 7, 9, 11\}$ , with only eighth forbidden transitions in general or four if we exclude the tones outside the scale. Thus, we may consider two types of mystic chord: one, like  $M$ , which shares most of its tones with the “even” whole-tone scale  $\{0, 2, 4, 6, 8, 10\}$ , and the other one that is closer to the “odd” whole-tone scale.

Hence, in general, for the even mystic chord, a very good scale for counterpoint is the odd whole-tone scale, and vice versa.

Unfortunately, for the whole-tone scale there is no analogue of Noll’s theorem connecting a harmony based on triads and counterpoint, since among all possible triads there is none whose set of endomorphisms is such that their linear part yields a strong dichotomy. This, by the way, is in accordance with classical musicological opinion on the scale of its poor harmonic possibilities (from the tonal harmony perspective, at least [3, p. 486]), and perhaps it was an attractive characteristic for Scriabin to use in his music.

## 2. SOME STATISTICAL CONTRAPUNTAL PROPERTIES OF THE FUXIAN AND MYSTIC WORLDS

In Table 1 we see the distribution of the number of contrapuntal symmetries between all intervals for the Fuxian and mystic worlds<sup>1</sup>. If you have, for instance, the step  $(0+\epsilon 3, 2+\epsilon 4)$  in the Fuxian world, there are two counterpoint symmetries that “allow” it. Contrariwise, the step  $(0+\epsilon 7, 2+\epsilon 7)$  is forbidden, for it has 0 counterpoint symmetries. In the Fuxian world the maximum number of symmetries mediating in a step is 5. For the  $12^4$  possible steps (dissonance to dissonance, dissonance to consonance and consonance to dissonance steps can be considered in the extended model), we have a total of 6720 inadmissible steps, 4992 steps with only one counterpoint symmetry, and so on within the Fuxian world. In Table 2 we find the mean and standard deviation of the distributions of Table 1.

The probability that for a random transition to be valid in the Fuxian world is  $p_F = \frac{14016}{12^4}$ , for the mystic world it is  $p_M = \frac{4608}{12^4}$ , whilst the probability of being valid in both worlds is  $p_{F \cap M} = \frac{2976}{12^4}$ . Thus the validity of a transition in these worlds are approximately independent events, since  $|p_{F \cap M} - p_F p_M| < \frac{1}{110}$ .

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<sup>1</sup>A *counterpoint world* is a directed graph, where each vertex is a counterpoint interval and there is an arc connecting valid steps. See [1, Chapter 4] for further details.

Number of counterpoint symmetries	Number of Fuxian steps	Number of mystic steps
0	6720	16128
1	4992	576
2	5568	2880
3	1440	0
4	1152	1152
5	864	0

TABLE 1. Distribution of the number of counterpoint symmetries for the Fuxian and mystic worlds.

Counterpoint world	Mean	Standard deviation
Fuxian	1.4167	1.3651
Mystic	0.5278	1.9026

TABLE 2. Mean and standard deviation of the number of counterpoint symmetries in steps within the Fuxian and mystic worlds.

### 3. FUXIAN AND MYSTIC WORLDS IN SCRIBIN'S PIANO SONATA NO. 5, OP. 53

We can find some connections of Mazzola's contrapuntal model with Scriabin's Piano Sonata No. 5, op. 53 [5], which is notable for the explicitness of the mystic chord.

In the introductory section that spans measures 13 to 31, taking the cantus firmus as E (as suggested by a standard musicological analysis of the work [7, pp. 2-3]), we count 30 contrapuntal transitions. If they are regarded as living within the Fuxian world, the mean number of counterpoint symmetries is 1.3333 with standard deviation 1.1244. The effect size of this sample with respect to the whole distribution is 0.061, with a 90% confidence interval  $[-0.239, 0.362]$ , that qualifies as a small. This means that, seen as Fuxian first species counterpoint, it is essentially indistinguishable from a random selection.

On the other hand, in the mystic world, the mean number of counterpoint symmetries is 2.1000, with standard deviation 1.5614. The effect size is now 1.438, with a 90% confidence interval  $[1.137, 1.739]$ , which indicates a positively large effect. In other words, considering the difficulty of finding valid steps in the mystic world, it is very noticeable the non-randomness of Scriabin elections of counterpoint intervals within it.

If we do a  $\chi^2$  test (with 5 degrees of freedom) for the frequencies of number of symmetries, we find the value of 7.83 for the statistic (with Yates's correction for continuity [6, p. 53]) in the Fuxian world; the  $p$ -value is 0.16575. In contrast, for the mystic world (with 3 degrees of freedom) the value of the statistic is 57.72 (also with correction for continuity), thus the  $p$ -value is  $1.8 \times 10^{-12}$ . This further confirms that the election of transitions can be seen as random in the Fuxian world and non-random in the mystic world, keeping in mind that the correlation of validity of steps between them is very small.

Repeating this analysis for measures from 47 to 67, we find 52 possible transitions (omitting repetitions of certain patterns). The Fuxian mean and standard deviations are 1.6731 and 0.87942, with a corresponding effect size of 0.188 with a 90% confidence interval of [0.04, 0.416], which is small but significant. For the mystic world, the mean and standard deviations are 0.69231 and 1.2294, with effect size of 0.151 and a 90% confidence interval of [0.078, 0.379], which is clearly smaller than the Fuxian one. This implies that the character of the piece clearly changes in its contrapuntal character, which becomes somewhat ambiguous. This conclusion is reinforced and clarified by the  $\chi^2$  test: the statistic for the Fuxian world is 36.385 with  $p$ -value of  $7.95 \times 10^{-7}$ , whereas for the mystic world the values are 0.57184 and 0.90285, respectively. In other words: while the first exposition of the sonata does not use functional tonal harmony, it is overwhelmingly closer to the counterpoint of the standard consonances than to one stemming from the mystic chord heard in the introduction.

#### 4. SOME ADDITIONAL OBSERVATIONS

While not as explicit as an extraction of first species counterpoint from the sonata, we can find more evidences of the importance of the mystic chord as a choice of consonances and the role the whole tone scale has with respect to it.

For instance, from measure 102 to 103 he favors pitches within the even whole-tone scale and in 104 and 105 he states an odd mystic chord; then suddenly changes the key and begins to stress the even whole-tone scale.

Another similar situation occurs in measures 130 and 131, where Scriabin displays an arpeggiated even mystic chord followed by an arpeggiated odd whole-tone scale in the following measure. Quite interestingly, this is continued by an odd mystic chord in measures 136 and 137, but preceding it with and ambiguity between the even and

odd whole tone scale, anticipating another sudden change of mood in measure 140.

A final explicit apparition of an even mystic chord in measure 262 is also associated with a dynamical fluctuation in the piece, but in this case its interaction with the whole scale is less apparent but seems to be in favor of the odd whole-tone scale, as expected.

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